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Matrix Theory Approach to Complex Waves

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***Abstract*—Complex waves in shielded lossless inhomogeneous isotropic guides are investigated. A critical appraisal of the existing theory of complex waves is given and new approach is proposed. A mathematical condition for the existence of complex waves is derived using the properties of a generalized symmetric matrix eigenvalue problem. It is shown that complex waves may exist in slightly perturbed homogeneous guides as a result of the coupling of a pair of degenerate or nearly degenerate modes.**

I. Introduction

Complex waves are the modes guided by shielded lossless guides which have complex propagation constants despite the lossless nature of the structure. A first theory of complex waves was published by Chorney as early as in 1961 [7], in a research report devoted to the properties of waves supported by anisotropic bidirectional guides. Complex waves do not exist in hollow cylindrical guides and initially it was believed that lossless shielded uniform dielectric guides can not generally support modes with complex propagation constants [6]. Pioneering work by Clarricoats and coworkers [14]–[16] proved that complex waves can be excited in a circular waveguide containing a coaxial dielectric rod. Similar result was obtained independently by Belyantsev and Gaponov [9] who discovered complex waves in coupled lines. Since then complex and backward waves in a circular waveguide containing a dielectric rod have been the subject of thorough numerical investigations [14]–[23]. It was established that complex waves may only be excited if the permittivity of the dielectric rod is high enough. It was also found that a complex wave carries no power and the existence of complex waves was finally confirmed experimentally [4]. These detailed studies published now and again in the literature were accompanied by theoretical consideration which gave deeper insight into the nature of complex waves [5], [8].

In the 1980's the scope of the research into inhomogeneous guides broadened and complex waves were reported for a variety of waveguiding structures including dielectric image guide [24]–[25], microstrip [27], [31], [32] and fin line [10], [31]. An interesting result was obtained by Omar and Schünemann [12], [13] who proved that although a single complex wave carries no power, two complex waves forming a pair are not orthogonal with respect to cross power and consequently a pair as a whole behaves as a mode below cutoff carrying purely reactive power. It was also found that in certain structures, for instance in a rectangular image guide investigated by Strube and Arndt [24], it is possible to obtain complex waves even if the perturbation caused by inhomogeneity is relatively small. The attention which complex waves have received recently is primarily due to the role which they play in the discontinuity analysis. Investigations have shown that complex waves constitute an essential part of modal spectrum and their omission may lead to erroneous results in certain discontinuity problems [12], [28]–[31].

The intensive studies into the complex waves resulted in 1987 in a paper [13] by Omar and Schünemann which was intended as a general treatment of complex waves. Omar and Schünemann's 1987 paper use the approach similar to the one used in Chorney's 1961 report. In both cases the original boundary value problem was converted into a matrix eigenvalue problem but using slightly different techniques (It can be shown [33] that the two approaches are identical for infinite matrix dimensions). Chorney concentrated his work on the derivation of integral relations for complex waves. Omar and Schünemann proposed using the symmetry of the characteristic matrix as a criterion for the existence of complex waves. In this contribution we will show that some of Omar and Schünemann's conclusions are premature and propose a more rigorous approach to complex waves.

II. Mathematical Formulation

As a departure point for the analysis we shall use the matrix formulation derived by Omar and Schünemann [13] who investigated a general lossless structure of a uniform waveguide inhomogeneously filled with a dielectric whose relative permittivity was a function of transverse coordinates ($\epsilon_r = \epsilon_r(r)$). The fields in the guide were assumed to have the z dependence in the form $e^{-j\beta z}$, and expanded in series of normalized longitudinal components of TM and TE modes existing in the empty waveguide [11], [13]. The expansion coefficients and the propagation constants of the modes supported by the loaded guide can be obtained from the following matrix eigenvalue equation (eq. 12 in [13]):

$$\begin{bmatrix} (k_0^2\underline{\underline{I}} - \underline{\underline{S}}^{-1})\underline{\underline{R}}^e & -\omega\mu_0/\lambda(k_0^2\underline{\underline{I}} - \underline{\underline{S}}^{-1})\underline{\underline{T}} \\ -\omega\epsilon_0\lambda\underline{\underline{T}}^T & k_0^2\underline{\underline{R}}^h - \underline{\underline{\Lambda}}^h \end{bmatrix} \begin{bmatrix} \underline{A}' \\ \underline{B}' \end{bmatrix} = \beta^2 \begin{bmatrix} \underline{A}' \\ \underline{B}' \end{bmatrix} \tag{1}$$

Manuscript received February 21, 1990; revised August 18, 1991. This work was supported by the British Council under the British Council Fellowship Scheme and the Polish Academy of Sciences under contract CPBP-02.02.3.2.

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IEEE Log Number 9106051.

where $\underline{\underline{I}}$ is the identity matrix T stands for the transposition sign, β, k_0, λ are the unknown propagation constant, the wave number of the free space ($k_0^2 = \omega^2\mu_0\epsilon_0$) and a real positive constant, respectively while $A' = A/\sqrt{\lambda}$, $B' = -j\sqrt{\lambda}\underline{B}$ are column vectors containing expansion coefficients. The elements of the matrices appearing in the above equation are defined in [13] by the equations (7). All matrices in (1) are real and have infinite dimensions. Additionally the matrices $\underline{\underline{R}}^e$, $\underline{\underline{R}}^h$, $\underline{\underline{S}}$ are symmetric.

Omar and Schünemann showed that for certain special cases a general eigenvalue problem given by (1) can be written in a simpler form. For instance, for modes showing no φ dependence supported by a circular waveguide containing a dielectric rod, equation (1) becomes (eqs. 29 in [13]):

$$[(k_0^2\underline{\underline{I}} - \underline{\underline{S}}^{(0)^{-1}})\underline{\underline{R}}^{e(0)}]\underline{A}^{(0)} = \beta^2\underline{A}^{(0)} \tag{2}$$

$$[k_0^2\underline{\underline{R}}^{h(0)} - \underline{\underline{\Lambda}}^{h(0)}]\underline{B}^{(0)} = \beta^2\underline{B}^{(0)} \tag{3}$$

where superscript 0 denotes no variation in the φ direction.

The theory of complex waves developed by Omar and Schünemann is based on the investigation of the symmetry of the characteristic matrices in each of the above eigenvalues problems. The characteristic matrix of the problem (1) is nonsymmetric and hence its eigenvalues β^2 can take complex values. For special cases, described by the equations (29) and (37) in [13], Omar and Schünemann state that the characteristic matrices are symmetric and accordingly no complex modes may exist. Unfortunately, although the conclusions formulated by Omar and Schünemann are apparently true, the reasoning used for their derivation can not be accepted. For instance first sentence following equation (12) in [13] reads "The two diagonal submatrices of the characteristic matrix in (12) are real and symmetric." The two diagonal submatrices are $(k_0^2\underline{\underline{I}} - \underline{\underline{S}}^{-1})\underline{\underline{R}}^e$ and $k_0^2\underline{\underline{R}}^h - \underline{\underline{\Lambda}}^h$. The latter is symmetric indeed but the former is a product of two symmetric matrices and its symmetry is ensured only when the component symmetric matrices commute. Omar and Schünemann do not investigate if the component matrices appearing in (12), (29), (37) in [13] are commutable. It can easily be proven that the product $\underline{\underline{\Lambda}}^e\underline{\underline{S}}$ appearing in (37) in [13] is not symmetric because the operation $\underline{\underline{\Lambda}}^e\underline{\underline{S}}$ multiplies each row of the symmetric matrix $\underline{\underline{S}}$ by a different constant and thus the symmetry of the resulting matrix is lost. Accordingly the reality of the modal spectrum could not have been inferred from the symmetry of the characteristic matrix because in most cases discussed in [13], the characteristic matrix was nonsymmetric. This proves that the conclusions drawn by Omar and Schünemann are premature. Furthermore, the criterion employed by Omar and Schünemann, the asymmetry of the characteristic matrix, is not subtle enough to distinguish problems with nonsymmetric matrix can lead to complex waves from those which can not.

Fortunately, in all cases discussed by Omar and Schünemann, the characteristic matrix is factorized into a product of two symmetric matrices. Consequently, each matrix eigenvalue problem with nonsymmetric matrix can be transformed to the form

$$\underline{\underline{A}}\underline{x} = \sigma\underline{\underline{M}}\underline{x} \tag{4}$$

with both $\underline{\underline{A}}$ and $\underline{\underline{M}}$ being symmetric matrices $\sigma = \beta^2$ denoting the eigenvalue. For instance, multiplying the first "row" of (1) by $\lambda^2\epsilon_0$ $[k_0^2\underline{\underline{I}} - \underline{\underline{S}}^{-1}]^{-1}$ (the inversion exists except for modes at cutoff) and the second "row" by μ_0 we obtain

$$\begin{bmatrix} \lambda^2\epsilon_0\underline{\underline{R}}^e & -\omega\mu_0\epsilon_0\lambda\underline{\underline{T}} \\ -\omega\mu_0\epsilon_0\lambda\underline{\underline{T}}^T & \mu_0(k_0^2\underline{\underline{R}}^h - \underline{\underline{\Lambda}}^h) \end{bmatrix}\begin{bmatrix} \underline{A}' \\ \underline{B}' \end{bmatrix} = \beta^2\begin{bmatrix} \lambda^2\epsilon_0[k_0^2\underline{\underline{I}} - \underline{\underline{S}}^{-1}]^{-1} & 0 \\ 0 & \mu_0\underline{\underline{I}} \end{bmatrix}\begin{bmatrix} \underline{A}' \\ \underline{B}' \end{bmatrix} \tag{5}$$

A. The Necessary Condition for the Existence of Complex Waves

Formula (4) describes a symmetric generalized matrix eigenvalue problem. It is a common error to assume that the symmetry of matrices in a generalized eigenvalue problem implies that the eigenvalues are real. We shall prove that the symmetry is not enough.

Each generalized matrix eigenvalue problem can be characterized by a pair of matrices $(\underline{\underline{A}}, \underline{\underline{M}})$. Such a pair is called a pencil [2]. The eigensolutions of a symmetric pencil have several interesting properties which can be used to derive the condition for the existence of complex waves.

Let us denote by σ_i, $\underline{x}_i$ and σ_j, $\underline{x}_j$ two eigensolutions (not necessarily distinct ones) of the pencil $(\underline{\underline{A}}, \underline{\underline{M}})$. Owing to the symmetry of the pencil the following relation may be obtained from (4) [33]:

$$(\sigma_i - \sigma_j^*)\langle \underline{\underline{M}}\underline{x}_j, \underline{x}_i\rangle_h = 0 \tag{6}$$

and for $\sigma_i, \sigma_j \neq 0$

$$(1/\sigma_i - 1/\sigma_j^*)\langle \underline{\underline{A}}\underline{x}_j, \underline{x}_i\rangle_h = 0 \tag{7}$$

where $\langle\cdot,\cdot\rangle_h$ denotes the hermitian inner product defined by

$$\langle \underline{y}, \underline{x}\rangle_h := \underline{y}^H\underline{x} \tag{8}$$

with $\underline{y}$, $\underline{x}$ being arbitrary column vectors and the superscript H denoting hermitian transposition.

Relations (6) and (7) state that distinct eigenvectors of a symmetric pencil $(\underline{\underline{A}}, \underline{\underline{M}})$ are M- and A-orthogonal. Forms $\underline{x}_i^H\underline{\underline{M}}\underline{x}_i$ and $\underline{x}_i^H\underline{\underline{A}}\underline{x}_i$ have physical interpretations. For instance it can be shown [33] that

$$\frac{\beta_j^* k_0}{2}\langle \underline{\underline{M}}\underline{x}_j, \underline{x}_i\rangle_h = \frac{1}{2}\int_S (\vec{E}_i \times \vec{H}_j^*)\cdot d\vec{S} \tag{9}$$

$$\frac{\beta_j^*\beta_i}{4}\langle \underline{\underline{M}}\underline{x}_j, \underline{x}_i\rangle_h = \frac{\mu_0}{4}\int_S (\vec{H}_{ti}\cdot\vec{H}_{tj}^*)\,dS - \frac{\epsilon_0}{4}\int_S (\vec{E}_{zi}\cdot\epsilon_r\vec{E}_{zj}^*)\,dS \tag{10}$$

$$\frac{1}{4}\langle \underline{\underline{A}}\underline{x}_j, \underline{x}_i\rangle_h = \lambda\left(\frac{\epsilon_0}{4}\int_S (\vec{E}_{ti}\cdot\epsilon_r\vec{E}_{tj}^*)\,dS - \frac{\mu_0}{4}\int_S (\vec{H}_{zi}\cdot\vec{H}_{zj}^*)\,dS\right). \tag{11}$$

Thus the hermitian forms $\underline{x}_i^H\underline{\underline{M}}\underline{x}_i$ and $\underline{x}_i^H\underline{\underline{A}}\underline{x}_i$ are interpreted as power carried by a mode or energy associated with selected field components. Using the above identities one may express orthogonality properties (6) and (7) in terms of integral relations between field components and derive the relation between backward and complex waves [33], [34]. Note, that for $i = j$ the hermitian forms $\underline{x}_i^H\underline{\underline{M}}\underline{x}_i = \langle \underline{\underline{M}}\underline{x}_i, \underline{x}_i\rangle_h$ and $\underline{x}_i^H\underline{\underline{A}}\underline{x}_i = \langle \underline{\underline{A}}\underline{x}_i, \underline{x}_i\rangle_h$ must vanish for any eigenvector corresponding to a complex eigenvalue in order for (6) and (7) to be fulfilled. Hence, the eigenvector $\underline{x}_i$ corresponding to a complex wave will give

$$\underline{x}_i^H\underline{\underline{A}}\underline{x}_i = 0 \quad\text{and}\quad \underline{x}_i^H\underline{\underline{M}}\underline{x}_i = 0. \tag{12}$$

In physical terms it means that integrals (9)–(11), expressing power carried by a mode or energy associated with selected field component, vanish for complex waves. Hence, the theory of symmetric pencils gives the result which agrees with earlier observations, namely that a complex wave carries no power. If either of the matrices in a pencil is definite the hermitian form $\underline{x}^H\underline{\underline{M}}\underline{x}$ or $\underline{x}^H\underline{\underline{A}}\underline{x}$ is either positive or negative and consequently no complex eigenvalues are allowed.

To sum up we observe that complex eigenvalues may occur only if *both* $\underline{\underline{M}}$ *and* $\underline{\underline{A}}$ are indefinite. The indefiniteness of both matrices in a pencil is merely a necessary condition for the existence of complex eigenvalues. On the other hand the definiteness of at least one of the matrices in the pencil is a sufficient condition for the reality of the spectrum.

B. Definiteness of the Pencils Describing Wave Propagation

Let us now investigate the definiteness of the matrices in different wave guidance problems. First of all let us observe that the elements of the matrices $\underline{\underline{R}}^e$, $\underline{\underline{R}}^h$, $\underline{\underline{S}}$ and $\underline{\underline{F}}^c$ of [13] can be written in the following generic form:

$$G_{nm} = \int_S w(s) f_n(s) f_m(s)\, ds \qquad w(s) > 0. \tag{13}$$

Since

$$\underline{x}^T\underline{\underline{G}}\underline{x} = \int_S w(s)\left(\sum_i x_i f_i(s)\right)^2 ds > 0 \tag{14}$$

these matrices are positive definite [1].

Let us first concentrate on the special cases described by (2)–(3). Problem (3) has a symmetric characteristic matrix and consequently, as Omar and Schünemann correctly observed no complex eigenvalues are allowed. Equation (2) has a nonsymmetric characteristic matrix but this problem can be written in the form of a symmetric pencil with

$$\underline{\underline{A}} = k_0^2\underline{\underline{I}} - \underline{\underline{S}}^{(0)^{-1}} \quad \text{and} \quad \underline{\underline{M}} - \underline{\underline{R}}^{e(0)^{-1}}. \tag{15}$$

Since $\underline{\underline{R}}^{e(0)}$ is positive definite then the form $\underline{x}_i^H\underline{\underline{M}}\underline{x}_i$ never vanishes and accordingly complex eigenvalues can not occur. As a result modes with no angular variation supported by a circular waveguide with a coaxial dielectric rod can not become complex. Similar result can be obtained for the rectangular waveguide with one dimensional inhomogeneity (eqs. 37 in [13]). Here we have two symmetric pencil with the matrices $\underline{\underline{M}} = [\underline{\underline{\Lambda}}^e]^{-1}$ and $\underline{\underline{M}} = (\underline{\underline{F}}^c)^{-1}$ both being positive definite.

We have demonstrated that for certain types of boundary value problems complex waves can never occur despite the asymmetry of the characteristic matrix. It now remains to show that in a general case both matrices in the pencil are indefinite. We shall use a theorem which states that a symmetric matrix is positive definite if and only if all its principal submatrices are positive definite [3]. Let us consider the matrix $\underline{\underline{A}}$. The principal submatrix $\underline{\underline{R}}^e$ is positive definite. We shall prove that the submatrix $k_0^2\underline{\underline{R}}^h - \underline{\underline{\Lambda}}^h$ contains a principal submatrix which is negative definite. Note that because the modes in the basis guide are normalized and $\epsilon_r > 0$, the elements on the main diagonal of matrix $\underline{\underline{R}}^h$ are bounded above:

$$\begin{aligned} R^h_{mm} &= 1/k^2_{mh} \int_S \epsilon_r |\nabla_t h_{zm}|^2\, dS \\ &\le \epsilon_{\max} \frac{1}{k^2_{mh}} \int_S |\nabla_t h_{zm}|^2\, dS = \epsilon_{\max} \end{aligned} \tag{16}$$

where $\epsilon_{\max}$ is the maximal value of permittivity of the inhomogeneous medium filling the guide. Consequently, as the mode index n in the submatrix $k_0^2\underline{\underline{R}}^h - \underline{\underline{\Lambda}}^h$ increases, the elements on the main diagonal of the matrix $\underline{\underline{R}}^h$ remain bounded while the elements in the matrix $\underline{\underline{\Lambda}}^h$ tend to $+\infty$. Therefore, if we pick a sufficiently large mode index N then the submatrix $[k_0^2\underline{\underline{R}}^h - \underline{\underline{\Lambda}}^h]_N$ consisting of all but first N modes will become diagonally dominant and will have negative diagonal elements. According to the Gerschgorin localization theorem [1], a symmetric, diagonally dominant matrix with negative diagonal elements is negative definite. Hence, the matrix $\underline{\underline{A}}$ contains both positive and negative definite principal submatrices which means that $\underline{\underline{A}}$ is indefinite. Similar proof can be given for matrix $\underline{\underline{M}}$. Here we have a positive definite matrix $\underline{\underline{I}}$ and a submatrix $[k_0^2\underline{\underline{I}} - \underline{\underline{S}}^{-1}]_N$ which becomes negative definite if a sufficiently large N is taken.

III. Slightly Perturbed Homogeneous Guides

A. Sensitivity of Basis Modes with Respect to the Creation of Complex Waves

We shall now apply the necessary condition for existence of complex eigenvalues $\underline{x}^H\underline{\underline{M}}\underline{x} = 0$ to determine which basis modes are liable to become complex if an unloaded basis guide supporting only purely TE and TM modes is perturbed by a slight inhomogeneity.

Let the unperturbed system be described by the pencil $(\underline{\underline{W}}, \underline{\underline{M}})$. Let η_i be a simple eigenvalue of the pencil $(\underline{\underline{W}}, \underline{\underline{M}})$ and $\underline{x}_i$ be its associated eigenvector. The spectrum of the pencil $(\underline{\underline{W}}, \underline{\underline{M}})$ is real. Hence,

$$\underline{x}^H\underline{\underline{M}}\underline{x} \neq 0. \tag{17}$$

We want now to find out if the complex eigenvalues will occur when the pencil $(\underline{\underline{W}}, \underline{\underline{M}})$ is perturbed. We will seek an approximation to the eigenvector $\underline{\tilde{x}}$ of the pencil $(\underline{\underline{A}}, \underline{\underline{M}}) = (\underline{\underline{W}} + \underline{\underline{K}}, \underline{\underline{M}})$ where $\underline{\underline{K}}$ is a fixed perturbation.

If we put $\epsilon = \|\underline{\underline{M}}^{-1}\underline{\underline{K}}\|$ then we have the following estimate [3]:

$$\|\underline{x}_i - \underline{\tilde{x}}_i\| \le \frac{\epsilon}{\delta} \tag{18}$$

where

$$\delta = \min_{i \neq j} |\eta_i - \eta_j|. \tag{19}$$

The above inequality shows that the eigenvectors corresponding to poorly separated eigenvalues of unperturbed system are ill conditioned and even a slight perturbation may cause a significant change in components of the eigenvector $\underline{\tilde{x}}$ compared with $\underline{x}$. Conversely, for well separated eigenvalues, the eigenvectors will be only slightly perturbed by a small inhomogeneity.

Suppose $\underline{x}$ is normalized so that $\underline{x}^H\underline{\underline{M}}\underline{x} = 1$. Then, for well separated eigenvalues of unperturbed pencil, it follows from (18) and (17) that

$$\underline{\tilde{x}}^H\underline{\underline{M}}\underline{\tilde{x}} \approx \underline{x}^H\underline{\underline{M}}\underline{x} \neq 0. \tag{20}$$

The same relation can not be written for the ill conditioned eigenvectors $\underline{\tilde{x}}$. Despite a small perturbation we may, in this case, obtain

$$\underline{\tilde{x}}^H\underline{\underline{M}}\underline{\tilde{x}} = 0 \tag{21}$$

which is the necessary condition to the occurrence of complex eigenvalues.

The eigenvalues of unperturbed pencil correspond to the squares of the propagation constants of the basis TE and TM modes. Thus, the degeneracy between basis modes brings about poorly separated eigenvalues of the pencil $(\underline{\underline{W}}, \underline{\underline{M}})$ and accordingly degenerate basis modes are particularly prone to become complex when the basis guide is perturbed by a dielectric insert. Using the physical interpretation of the necessary condition one may prove [33] that vanishing of the hermitian form $\underline{\tilde{x}}^H\underline{\underline{M}}\underline{\tilde{x}}$ may take place only when the degeneracy occurs between basis modes of the opposite type i.e., TE-TM. Hence, the pairs of the degenerate TE-TM basis modes are potentially capable of creating complex waves when a basis structure is perturbed with a dielectric insert.

The results of the perturbation analysis show that investigating complex waves excited by a small perturbation we can restrict ourselves to the analysis of the coupling between a pair of the nearest TE and TM basis modes. For this case the size of the matrix in (1) is reduced to 2 and from the resulting characteristic equation one can obtain the analytical condition for existence the complex waves [33], [35]. Investigating these conditions one arrives at the conclusion that in a slightly perturbed homogeneous guide a complex wave occurs as a result of a coupling of the degenerate or nearly degenerate cutoff TE-TM basis modes which are not orthogonal in a sense of the integral

$$\int_{S_0} (\epsilon_r - 1)(\nabla_t e_r \times \nabla_t h_z) \cdot d\vec{S} \neq 0 \tag{22}$$

where S_0 is the perturbed region.

B. Guide Geometries and Modes Prone to Complex Waves

The degeneracy between basis modes of different type is a crucial factor in the creation of complex modes in sightly inhomogeneous guides. We may therefore assume that the geometries in which such degeneracies frequently occur will be particularly prone to complex waves. One example of such a geometry is a rectangular waveguide in which all E_{nm} modes are degenerate, forming pairs with the H_{nm} modes. Degeneracies also occur in a circular waveguide. Here, the degeneracy takes place between H_{0m} and E_{1m} modes. We have shown that the degeneracy is not a sufficient condition. Modes can not be orthogonal in the sense of integral (22). Because in the circular guide the degenerate modes in one pair have different angular dependence, the perturbation in the form of a coaxial rod will not cause the necessary coupling between fields of basis modes, expressed by (32), and consequently such a configuration will be particularly robust. For offset rods this will no longer apply.

We can also draw an important practical conclusion regarding fundamental modes. In homogeneous structures a fundamental mode is not degenerate unless a guide exhibits certain symmetry; but then the degeneracy occurs for the modes of the same type, e.g., H_{10} and H_{01} in a square waveguide. Therefore dominant modes are intrinsically robust and will not easily yield complex waves. The same is true for the higher order modes in a rectangular guide which do not have the counterpart of the opposite type.

Obviously for large perturbations, introduced by high permittivity dielectrics the approximate bi-mode analysis is invalid because coupling with other not degenerate modes may prove decisive and result in the excitation or suppression of a complex wave. Nevertheless even in these cases our theory indicates modes which should be considered first as candidates for the creation of complex waves.

Because of the limited length of the short paper the verification of the conclusions regarding complex waves in slightly perturbed guides is not given in this text. Additional material, including verification of the conclusions in this paper, approximate formulae for the complex wave range and the orthogonality relations for complex waves is contained in the conference papers [34], [35].

Acknowledgment

One author of this paper, M. Mrozowski wishes to thank Prof. J. B. Davies, Dr. A. Fernandez and Dr. R. Ettinger from the University College London for their support and assistance during his stay in London. The authors also wish to thank Mr. T. Jabłoński of the Polish Academy of Sciences in Warsaw for helpful criticism.

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